INITIAL STAGES OF THE ELECTRICAL EXPLOSION OF A WIRE IN AN LC-LOOP

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INTRODUCTION

A great amount of work has been devoted to the investigation of the phenomenon of the electrical explosion of a wire, and several models have been proposed to explain its physical mechanism (for example, [1-8]). The fact that there is no single generally accepted point of view indicates that with respect to the phenomenon of the electrical explosion of a wire, all is not yet clear. However, it cannot be disputed that the essence of the phenomenon of the electrical explosion of a wire is competition between two processes: the breakdown of the wire as a whole and the accompanying loss of electrical conductivity, on the one hand, and the evolution of Joule heat, on the other hand. Here the mechanism of the breakdown itself is determined by the rate of introduction of energy. With a very small rate of introduction, the breakdown of the wire takes place after its melting, as a result of the development of MHD instabilities. This process is comparatively slow (several hundred microseconds). If an energy corresponding to the point of its equilibrium boiling (or even greater) is introduced into the wire in considerably less time, new (more rapid) breakdown factors appear. Such a factor in [1-3] is the explosive volumetric boiling of a superheated liquid. In an MHD model [4], the loss of electrical conductivity is connected with the formation of constrictions, dividing the liquid wire into individual disks (strata). In [5], a model consisting of surface vaporization waves, running from the periphery of the exploding wire to its center, is proposed. Each of these processes has its characteristic time. With a still greater rate of heating, there can be introduced into the wire an amount of energy sufficient for its total vaporization (or even an amount of energy corresponding to its critical state) before any of these factors can manifest itself [1-5]. For this case of an electrical explosion a model of a "metallized" plasma has been proposed [6]. With high rates of rise in the current and considerable diameters of the wire, there is the possibility (in principle) of conditions of the explosion of a skin-layer [7]. In [8] the problem of the electrical explosion of a wire was solved on the basis of the equation of state of copper. With this approach, the density (and, consequently, also the conductivity) of the metal of the exploding wire with a given specific energy is the greater, the greater the rate of introduction of energy.

From what has been said it is clear that the moment of an electrical explosion and its physical mechanism are determined by the rate of introduction of energy. In [3], there was observed a change in the character of an electrical explosion as a function of the rate of heating of the liquid phase. The present article poses the problem of the determination of this rate from the initial conditions of the experiments.

1. Model of Quasi-Steady-State Heating

This model is the simplest and is based on the fact that the resistance of the wire is determined only by the value of the energy introduced into it due to Joule heating. From it there follows [9], where an analytical solution was found for the current and resistance of the exploding wire. This solution, however, is suitable only for the solid phase and for a time less than a quarter of the period of the discharge current. The same model was used in [10] for deriving equations describing the solid stage and melting. All of the stages of the electrical explosion of a wire can be calculated using the model of [8] by integration of the equations of hydrodynamics. In the present article, the relative (in fractions of the period of the LC-loop) duration of the three initial stages is found: solid, transitional (melting), and liquid [heating of the melt up to the boiling point as a function of two dimensionless parameters (the Q-factor q and the initial energy Q_0^*)]. Knowing the period of the loop T, we

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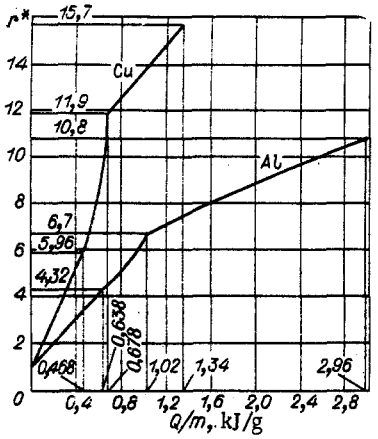

Fig. 1

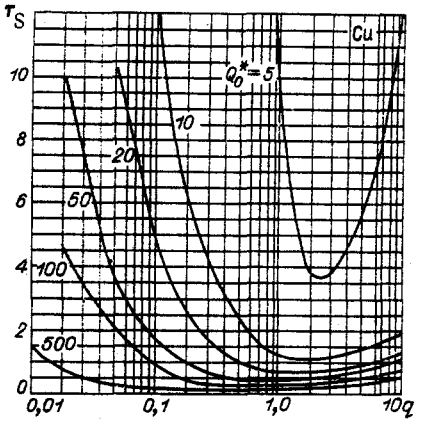

Fig. 2

can go over from the relative duration to the dimensional, i.e., we can determine the rate of introduction of energy, and this makes it possible to determine at what (approximately) moment of time and due to what factor the model of quasi-steady-state heating breaks down with given initial conditions. The exact determination of the breakdown of the model of quasi-steady-state heating must be carried out experimentally in each actual case. With exactly the same values of q and Q_0^*, this breakdown can set in different stages, depending on T: starting from the moment immediately after the melting point (with large values of T) and ending with the time after the melting point, that corresponds to superheating of the melt (with small values of T). Up to the moment of breakdown, the model of quasi-steady-state heating gives an approximate description of the initial stages of the electrical explosion of a wire and makes it possible to calculate them very simply before setting up an experiment. This possibility is also of importance in practice, since at the present time there are two known methods for a computer calculation [8, 11]; however, the first is rather complicated, and the second describes only the concluding stage.

2. Equations and Dimensionless Criteria of Process of Quasi-Steady-State Heating

The process of quasi-steady-state heating of a wire in an LC-loop is described by the equations

$$U_c = Li' + ri,\ U_c = U_0 - \frac{1}{C}\int_0^t idt,$$

$$r = r_0 f(Q),\quad Q' = ri^2,$$

where U_C is the voltage in the condenser; $U_0 = U_C|_{t=0}$; i is the current in the circuit; L is the total inductance in the loop (we assume it to be constant); r is the resistance of the exploding wire (the resistance of the loop is taken equal to zero); $r_0 = r|_{t=0}$; Q is the energy absorbed by the wire; a prime denotes differentiation with respect to the time. We introduce into the discussion the following dimensionless quantities: $\tau = t/\sqrt{LC}$; the functions $i^* = i/i_0$, $U^* = U_C/U_0$, and $r^* = r/r_0$ (a dimensionless function determining the rise of the resistance with the energy); and the two parameters $q = \rho/r_0$ (the Q-factor) and $Q_0^* = \beta CU_0^2/(2m)$ (the initial energy); here $i_0 = U_0/\rho$; $\rho = \sqrt{L/C}$; β is the thermal coefficient of the resistance of the wire in the solid stage; and m is the mass of the wire.

The system of equations is then brought into the form

$$di^*/d\tau = U^* - (1/q)r^*i^*; \tag{2.1}$$

$$dU^*/d\tau = -i^*; \tag{2.2}$$

$$dr^*/d\tau = F(r^*)r^*i^{*2}. \tag{2.3}$$

Equation (2.3) was obtained by differentiation of r* with respect to the time with a transition to dimensionless quantities and the introduction of the notation $F(r^*) = r_0 i_0^2 \sqrt{LC} r^{*\prime}$ [since Q is expressed in terms of r* from the equality $r^* = f(Q)$]. The dependences $r^*(Q)$ for the metals Cu and Al, plotted in accordance with the data of [12], are given in Fig. 1. The numerical integration of Eqs. (2.1)-(2.3) was done on an electronic computer. For the solid and transitional stages, a modified Euler method with an iterational analysis of each value

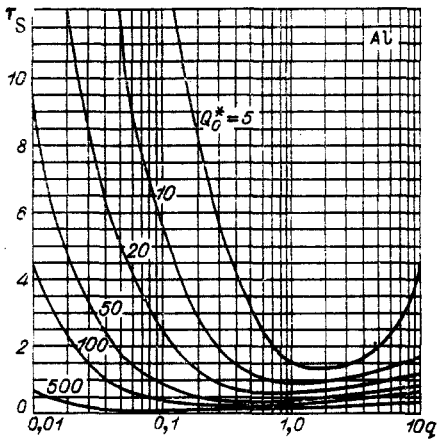

Fig. 3

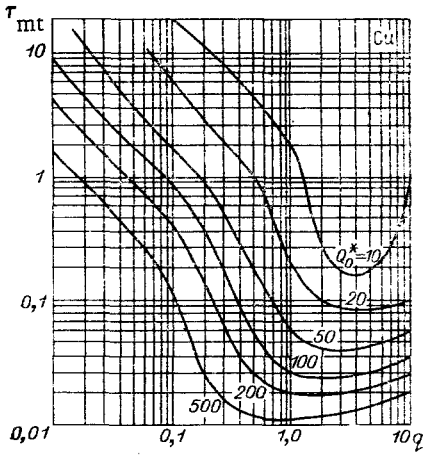

Fig. 4

of the function was used. For the liquid stage, use was made of a programmed KESSA complex, designed for the analysis of electronic schemes and involving an implicit method of integration [13]. The condition for stopping the calculation was the attainment of a value of r* corresponding to the end of the given stage.

3. Heating of Solid Phase up to Melting Point

Neglecting the insignificant (within the limits of 2%) effect of a change in the dimensions of the wire with the temperature on the value of the resistance, for f(Q) we obtain the expression

$$r^* = 1 + \beta Q/m,$$

where $\beta = \alpha/c_p$ is the thermal coefficient and α the temperature coefficient of the resistance; c_p is the specific heat capacity of the metal (the mean for the temperature interval of the solid stage). We have

$$F(r^*) = r_0 i_0^2 \sqrt{LC}\,\frac{\beta}{m} = 2\frac{Q_0^*}{q},$$
$$\frac{dr^*}{d\tau} = 2\frac{Q_0^*}{q} r^* i^{*2}. \qquad (3.1)$$

From Eqs. (2.1)-(2.3) and (3.1) it can be seen that the process is determined by the two dimensionless parameters q and Q_0^*. The dimensionless duration of the solid stage τ_S, found as a result of the numerical integration of Eqs. (2.1), (2.2), and (3.1) for Cu and Al, is shown as a function of q and Q_0^* in Figs. 2 and 3. The minimum of τ_S (i.e., the maximum of the mean power of the Joule heat) depends on Q_0^* and lies in the interval from q = 0.2 to q = 2. In other words, the power is maximal in the case where the resistance of the exploding wire is equal in order of magnitude to the wave resistance of the loop.

4. Melting

Assuming a model of surface melting, and assuming the solid and liquid phases to be wires connected in parallel, for the resistance we obtain

$$r^* = r_S^* \left\{1 - \left[1 - \frac{\rho_S}{\rho_L}\left(1 + \frac{\Delta V}{V}\right)\right]\frac{Q}{L_t m}\right\}^{-1}, \qquad (4.1)$$

where $r_S^* = r_S/r_0$; r_S is the resistance of the solid wire at the melting point; ρ_S and ρ_L are the specific resistances of the phases at the melting point; L_t and $\Delta V/V$ are the specific heat and the relative increase in the volume with a phase transition; and Q is the heat evolved in the melting stage. From (4.1) we find

$$F(r^*) = 2\frac{Q_0^*}{q r_S^* \beta L_t}\left[1 - \frac{\rho_S}{\rho_L}\left(1 + \frac{\Delta V}{V}\right)\right] r^{*2},$$
$$\frac{dr^*}{d\tau} = 2\frac{Q_0^*}{q r_S^* \beta L_t}\left[1 - \frac{\rho_S}{\rho_L}\left(1 + \frac{\Delta V}{V}\right)\right] r^{*3} i^{*2}. \qquad (4.2)$$

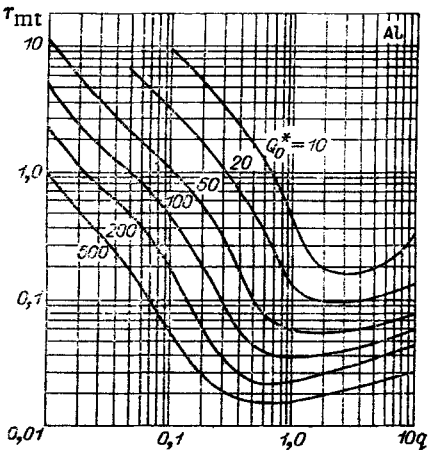

Fig. 5

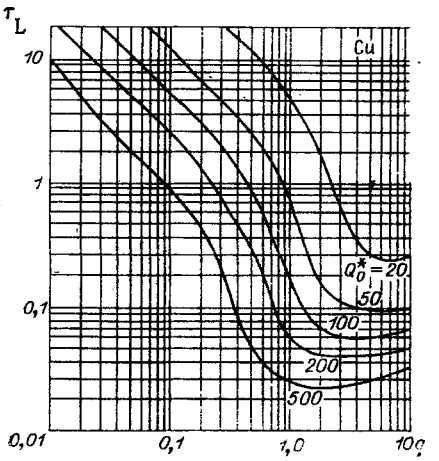

Fig. 6

For Cu, $r_S^* = 5.96$, $\rho_S/\rho_L = 0.48$, $L_t = 0.21$ kJ/g, $\Delta V/V = 4.5\%$, $\beta = 10.5$ g/kJ,

$$r^* = \frac{5.96}{1 - 2.37Q/m}(Q/m,\ \text{kJ/g}),$$

$$\frac{dr^*}{d\tau} = 0{,}076\frac{Q_0^*}{q}r^{*3}i^{*2}.$$

For Al, $r_S^* = 4.32$, $\rho_S/\rho_L = 0.61$, $L_t = 0.38$ kJ/g, $\Delta V/V = 6.6\%$, $\beta = 5.2$ g/kJ,

$$r^* = \frac{4.32}{1 - 0.92Q/m},\quad \frac{dr^*}{d\tau} = 0.082\frac{Q_0^*}{q}r^{*3}i^{*2}.$$

The duration of the melting stage τ_{mt} as a function of the Q-factor q and the initial energy Q_0^* (Figs. 4 and 5) was determined by integration of Eqs. (2.1), (2.2), and (4.2).

5. Heating of Liquid Phase up to Boiling Point

In this stage the effect of the temperature change in the dimensions of the exploding wire is considerable. We shall assume that the length of the wire is unchanged and that the increase in the volume takes place through the transverse cross section; the expression for r* can then be written in the form

$$r^* = r_L^*\left(1 + \frac{\beta_\rho/\beta_V - 1}{1 + \frac{1}{\beta_V Q/m}}\right),$$

where $\beta_\rho = \alpha_\rho/c_p$; $\beta_V = \alpha_V/c_p$; r_L is the resistance of the liquid wire at the melting point; α_V is the coefficient of volumetric expansion of the liquid phase; and α_ρ is the temperature coefficient of ρ.

From this we have

$$F(r^*) = 2\frac{Q_0^*}{q}\frac{\beta_V}{\beta}\frac{r_L^*}{\beta_\rho/\beta_V - 1}\left(\frac{\beta_\rho}{\beta_V} - \frac{r^*}{r_L^*}\right)^2,$$

$$\frac{dr^*}{d\tau} = 2\frac{Q_0^*}{q}\frac{\beta_V}{\beta}\frac{r_L^*}{\beta_\rho/\beta_V - 1}\left(\frac{\beta_\rho}{\beta_V} - \frac{r^*}{r_L^*}\right)^2 r^{*}i^{*2}. \qquad (5.1)$$

For Cu

$$\text{Cu}\quad r_L^* = 11.9,\quad \beta_\rho = \frac{\alpha_\rho}{c_p} = \frac{0.38\cdot10^{-3}}{0.47\cdot10^3} = 0.81\ \text{g/kJ},$$

$$\beta_V = \frac{\alpha_V}{c_p} = \frac{1.28\cdot10^{-4}}{0.47\cdot10^3} = 0.272\ \text{g/kJ},$$

$$r^* = 11.9\left(1 + \frac{1.98}{1 + \frac{1}{0.272Q/m}}\right),$$

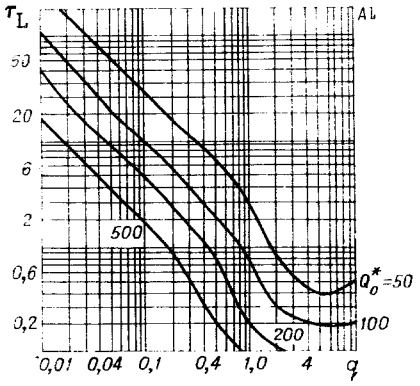

Fig. 7

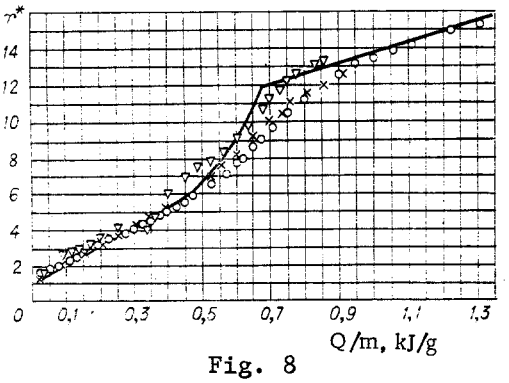

Fig. 8

$$\frac{dr^*}{d\tau} = 0.311\frac{Q_0^*}{q}\left(2.98 - \frac{r^*}{11.9}\right)^2 r^* i^{*2}.$$

For Al

$$\text{Al}\quad r_L^* = 6.7,\quad \beta_\rho = \frac{0.53\cdot10^{-3}}{1.08\cdot10^3} = 0{,}49\ \text{g/kJ},$$

$$\beta_V = \frac{1.2\cdot10^{-4}}{1.08\cdot10^3} = 0.111\ \text{g/kJ},$$

$$r^* = 6{,}7\left(1 + \frac{3.41}{1 + \frac{1}{0.111Q/m}}\right),$$

$$\frac{dr^*}{d\tau} = 0.084\frac{Q_0^*}{q}\left(4.41 - \frac{r^*}{6.7}\right) r^* i^{*2}.$$

Here $\alpha_V = 1.28\cdot10^{-4}$ 1/°C for Cu was taken from [10], and $\alpha_V = 1.2\cdot10^{-4}$ 1/°C for Al was calculated from the data of [14] from the dependence of the density of liquid Al on the temperature. The duration of the liquid stage τ_L, found by integration of (2.1), (2.2), and (5.1), is shown in Figs. 6 and 7.

6. Deviations of the Initial Stages of the Electrical Explosion of a Wire from the Model of Quasi-Steady-State Heating

Deviations arise both with too rapid, as well as with too slow, introduction of energy. In the first case they are connected with the appearance of temperature gradients over the cross section of the exploding wire as a result of the flow of heat to the phase interface during the melting period. According to [15], the value of the deviation is determined by the ratio δ^2 of the relaxation time of the temperature h_0^2/χ to the time of a phase transition with quasi-steady-state heating $L_t n/(f_t^2\rho_t)$, where h_0 is the thickness of the foil; χ is the thermal diffusivity; n is the specific density; and f_t^2 and ρ_t are the mean (for the melting stage) values of the square of the current density and the specific resistance, It is shown in [15] that for $\delta^2 = 1.9$ the process is very close to quasi-steady-state heating ($\delta^2 \to 0$), while for $\delta^2 = 30$ it differs very little from the limiting value ($\delta^2 \to \infty$). A limitingly rapid introduction of energy corresponds to a situation in which the dependence of the resistance of the exploding wire r* on Q/m in the "melting stage"† becomes linear and is a prolongation of the linear dependence for the solid stage. Since the dependence of r* on Q/m for Al in the melting stage (see Fig. 1) differs only slightly from the linear part corresponding to the solid stage, in this case the rate of introduction of energy will have no significant effect on the process up to the boiling point. This is confirmed by a comparison of calculated and experimental data made in [16]: With an insufficiently rapid introduction of energy, the curves of r*(Q/m) differ only insignificantly. For Cu this effect should be stronger, but, as can be seen from Fig. 1, with moderate values of the rate of introduction ($\delta^2 \cong 15$) deviations from the model of quasi-steady-state heating of ~16% are to be expected. In addition, the shift of the curve of r*(Q/m) downward from quasi-steady-state heating, due to a rate

†In this case melting does not take place, in spite of the sufficient value of the energy introduced, since the substance is not able to expand and decrease its density. Such a state of a "heated crystal" is metastable.

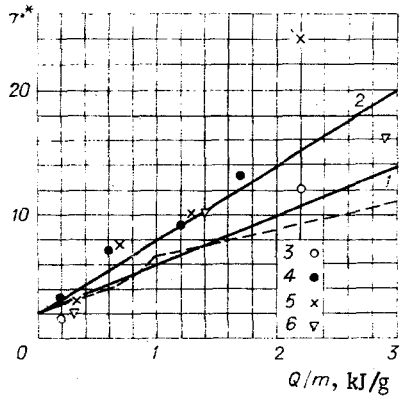

Fig. 9

of introduction of energy differing from zero, should be compensated to a certain degree by the instability of the phase interface [15]. Figure 8 gives calculated (according to a model of quasi-steady-state heating) and experimental results for Cu right up to the melting point. An electrical explosion of a foil with a thickness of $43 \cdot 10^{-6}$ m was carried out in an LC-loop with a period of $T = 6.7 \cdot 10^{-6}$ sec. The calculated curve was plotted from the starting data, and the experimental points were obtained from an analysis of oscillograms of the current and the voltage drop in the foil. It can be seen that the greatest deviations from the model of quasi-steady-state heating are actually observed in the transitional stage; in the solid and liquid stages the deviations are not great.

The deviations from the model of quasi-steady-state heating with slow introduction of energy are connected with the development of bending MHD instabilities [3, 4] and with the nonuniform nature of the heating of the wire up to the melting point due to the presence of inhomogeneities of structural and mechanical origin in the wire [3]. These inhomogeneities can lead to local volumetric melting and vapor formation and, as noted in [16], have a tendency to grow across the lines of the current. Figure 9 gives the dependences $r^*(Q/m)$ for aluminum foil with a thickness of 10^{-5} m, corresponding to the attainment of the melting point after a time of 0.9 and 1.5 μsec (curve 1 and 2, respectively). These curves were obtained by averaging over a series of experiments under exactly the same conditions. The scatter of the data was within the limits of the accuracy of the measurements (~10%). The lower the rate of introduction of energy, the greater the scatter of the data. In Fig. 9, points 3-6 denote four different experiments under exactly the same conditions. There was a sharp rise in the resistance before the attainment of the melting point at the moment of time $t \cong 13$ μsec, and there was a characteristic surge on the oscillogram of the voltage. It can be seen from these experiments that the slower the energy is introduced, the greater the degree to which the resistance of the exploding wire is determined by the development of inhomogeneities Furthermore, since the initial inhomogeneities and their development obviously have a random character, the scatter of the experimental data is increased.

A third factor which can bring about deviations from the model of quasi-steady-state heating is the skin effect. It is well known that the skin effect is practically absent with the condition $d \ll \delta_s$, where d is the diameter (or thickness) of the wire and $\delta_s = \sqrt{2/(\sigma\mu\omega)}$ is the thickness of the skin layer. In ordinary experiments on the electrical explosion of a wire this condition is satisfied [4].

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PHASE VELOCITIES AND DISCONTINUOUS STRUCTURE OF SHOCK WAVES

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UDC 533.951

§1. The problem of the structure of a shock wave consists in the search for the solution $u_k(x, t) = u_k(x - Ut)$ $(k = 1, \ldots, n)$ of a system of quasilinear equations of the form

$$\frac{\partial}{\partial t} A_i(\mathbf{u}) + \frac{\partial}{\partial x}\left[B_i(\mathbf{u}) - \sum_{k=1}^{m} \alpha_k C_{ik}(\mathbf{u}) \frac{\partial u_k}{\partial x}\right] = 0, \quad i = 1, \ldots, m, \tag{1.1}$$

$$\frac{\partial}{\partial t} A_i(\mathbf{u}) + \frac{\partial}{\partial x} B_i(\mathbf{u}) = 0, \quad i = m+1, \ldots, n,$$

with the boundary conditions $du_k/dx|_{x=\pm\infty} = 0$, where $\mathbf{u} = \{u_k\}_1^n$ is the set of parameters characterizing the state of the medium and $\{\alpha_k\}_1^m$ are the dissipative coefficients.

The extreme complexity of this problem, which arises largely due to the possibility of existence of segments of irregular behavior of the solution, does not permit the solution in its general formulation. At the same time, the determination and elimination of the irregular segments can appreciably simplify the problem. Such irregularities appear in the form of nonphysical segments in the solution, which generally correspond to regions of multivaluedness of some functions $u_k(x)$. The nonphysical segment in a formal mathematical solution must be replaced by a discontinuity, as usually done in hydrodynamics or magnetohydrodynamics in the study of shock waves [1-5]. It has been noted in a number of studies that an internal discontinuity appears in the case when the flow velocity in the wave goes through a certain critical value [2, 6-9]. Furthermore, it is shown that the critical velocity is the phase

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